Coordinate geometry is geometry dealing primarily with the line graphs and the (x, y) coordinate plane. The ACT Math Test includes nine questions on coordinate geometry. The topics you need to know are:

1. Number Lines and Inequalities
2. The (x,y) Coordinate Plane
3. Distance and Midpoints
4. Slope
5. Parallel and Perpendicular Lines
6. The Equation of a Line
7. Graphing Equations
8. Conic Sections

Most of the questions on coordinate geometry focus on slope. About two questions on each test will cover number lines and inequalities. The other topics are usually covered by just one question, if they are covered at all.

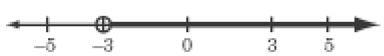
Number Lines and Inequalities

Number line questions generally ask you to graph inequalities. A typical number line graph question will ask you:

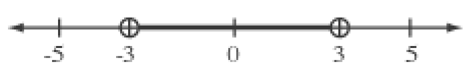
What is the graph of the solution set for $2(x + 5) > 4$?

To answer this question, you first must solve for x.

1. Divide both sides by 2 to get: $x + 5 > 2$
2. Subtract 5 from both sides to get: $x > -3$
3. Now you match $x > -3$ to its line graph:

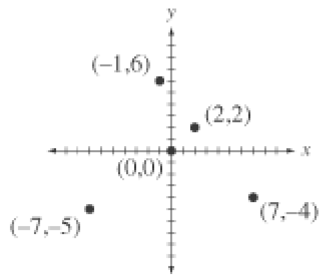


The circles at the endpoints of a line indicating an inequality are very important when trying to match an inequality to a line graph. An open circle at –3 denotes that x is greater than but *not* equal to –3. A closed circle would have indicated that x is greater than *or* equal to –3.
For the solution set $-3 < x < 3$, where x must be greater than –3 and less than 3, the line graph looks like this:



The (x,y) Coordinate Plane

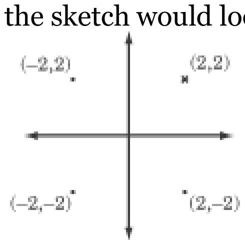
The (x,y) coordinate plane is described by two perpendicular lines, the x-axis and the y-axis. The intersection of these axes is called the origin. The location of any other point on the plane (which extends in all directions without limit) can be described by a pair of coordinates. Here is a figure of the coordinate plane with a few points drawn in and labeled with their coordinates:



As you can see from the figure, each of the points on the coordinate plane receives a pair of coordinates: (*x*,*y*). The first coordinate in a coordinate pair is called the *x*-coordinate. The *x*-coordinate of a point is its location along the *x*-axis and can be determined by the point's distance from the *y*-axis (*x* = 0 at the *y*-axis). If the point is to the right of the *y*-axis, its *x*-coordinate is positive, and if the point is to the left of the *y*-axis, its *x*-coordinate is negative.
The second coordinate in a coordinate pair is the *y*-coordinate. The *y*-coordinate of a point is its location along the *y*-axis and can be calculated as the distance from that point to the *x*-axis. If the point is above the *x*-axis, its *y*-coordinate is positive; if the point is below the *x*-axis, its *y*-coordinate is negative.
The ACT often tests your understanding of the coordinate plane and coordinates by telling you the coordinates of the vertices of a defined geometric shape like a square, and asking you to pick the coordinates of the last vertex. For example:

In the standard (*x*,*y*) coordinate plane, 3 corners of a square are (2,–2), (–2,–2), and (–2,2). What are the coordinates of the square's fourth corner?

The best way to solve this sort of problem is to draw a quick sketch of the coordinate plane and the coordinates given. You'll then be able to see the shape described and pick out the coordinates of the final vertex from the image. In this case, the sketch would look like this:



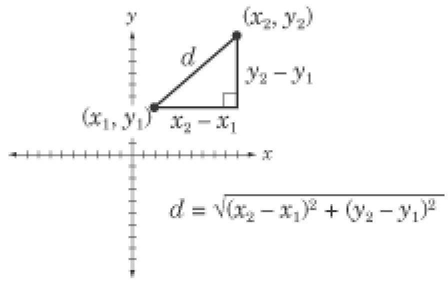
A square is the easiest geometric shape which a question might concern. It is possible that you will be asked to deal with rectangles or right triangles. The method for any geometric shape is the same, though. Sketch it out so you can see it.

Distance

The ACT occasionally asks test takers to measure the distance between two points on the coordinate plane. Luckily, measuring distance in the coordinate plane is made easy thanks to the Pythagorean theorem. If you are given two points, (x_1, y_1) and (x_2, y_2), their distance will always be given by the following formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between two points can be represented by the hypotenuse of a right triangle whose legs are of lengths $(x_2 - x_1)$ and $(y_2 - y_1)$. The following diagram shows how the formula is based on the Pythagorean theorem (see p.).



Here's a sample problem:

Calculate the distance between (4,–3) and (–3,8).

To solve this problem, just plug the proper numbers into the distance formula:

$$\text{Distance} = \sqrt{((-3)-4)^2 + (8-(-3))^2} = \sqrt{49+121} = \sqrt{170}$$

The distance between the points is $\sqrt{170}$, which equals approximately 13.04.

Finding Midpoints

Like finding the distance between two points, the midpoint between two points in the coordinate plane can be calculated using a formula. If the endpoints of a line segment are (x_1, y_1) and (x_2, y_2), then the midpoint of the line segment is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In other words, the *x*- and *y*-coordinates of the midpoint are the averages of the *x*- and *y*-coordinates of the endpoints.

Here is a practice question:

What is the midpoint of the line segment whose endpoints are (6,0) and (3,7)?

All you have to do is plug the end points into the midpoint formula. According to the question, $x_1 = 6, y_1 = 0, x_2 = 3,$ and $y_2 = 7$:

$$\text{Midpoint} = \left(\frac{6+3}{2}, \frac{0+7}{2}\right) = \left(\frac{9}{2}, \frac{7}{2}\right) = (4.5, 3.5)$$

Slope

The slope of a line is a measurement of how steeply the line climbs or falls as it moves from left to right. More technically, the slope is a line's vertical change divided by its horizontal change, also known as "rise over run." Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope of that line can be calculated using the following formula:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

The variable most often used to represent slope is *m*.

So, for example, the slope of a line that contains the points (–2,–4) and (6,1) is:

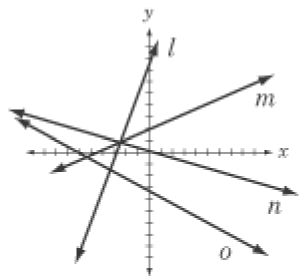
$$m = \frac{1-(-4)}{6-(-2)} = \frac{5}{8}$$

Positive and Negative Slopes

You can easily determine whether the slope of a line is positive or negative just by looking at the line. If a line slopes uphill as you trace it from left to right, the slope is positive. If a line slopes downhill as you trace it from left to right, the slope is negative.

You can determine the relative magnitude of the slope by the steepness of the line. The steeper the line, the more the "rise" will exceed the "run," and the larger $y_2 - y_1$ and, consequently, the slope will be. Conversely, the flatter the line, the smaller the slope will be.

For practice, look at the lines in the figure below and try to determine whether their slopes are positive or negative and which have greater relative slopes:

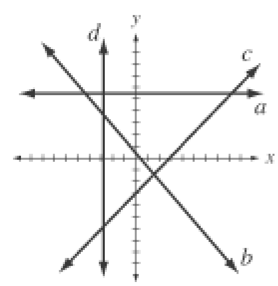


Lines *l* and *m* have positive slopes, and lines *n* and *o* have negative slope. In terms of slope magnitude, line *l* > *m* > *n* > *o*.

Special Slopes

It can be helpful to recognize a few slopes by sight.

- A line that is horizontal has a slope of 0. Since there is no "rise," $y_2 - y_1 = 0$, and thus $m = (y_2 - y_1)/(x_2 - x_1) = 0/(x_2 - x_1) = 0$.
- A line that is vertical has an undefined slope. In this case, there is no "run," and $x_2 - x_1 = 0$. Thus $m = (y_2 - y_1)/(x_2 - x_1) = ((y_2 - y_1)/0)$, and any fraction with 0 in its denominator is, by definition, undefined.
- A line that makes a $45°$ angle with a horizontal has a slope of 1 or –1. This makes sense because the "rise" equals the "run," and $y_2 - y_1 = x_2 - x_1$ or $y_2 - y_1 = -(x_2 - x_1)$.



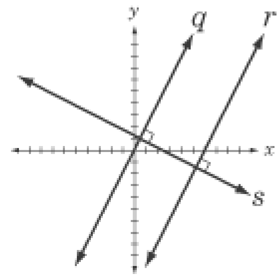
Line *a* has slope 0 because it is horizontal. Line *b* has slope –1 because it makes a $45°$ angle with the horizontal and slopes downward as you move from left to right. Line *c* has slope 1 because it makes a $45°$ angle with the horizontal and slopes upward as you move from left to right. Line *d* has undefined slope because it is vertical.

Parallel and Perpendicular Lines

Parallel lines are lines that don't intersect. In other words, parallel lines are lines that share the exact same slope.

Perpendicular lines are lines that intersect at a right angle (or 90%). In coordinate geometry, perpendicular lines have negative reciprocal slopes. That is, a line with slope m is perpendicular to a line with a slope of $-1/m$.

In the figure below are three lines. Lines q and r both have a slope of 2, so they are parallel. Line s is perpendicular to both lines q and r, and thus has a slope of $-1/2$.



On the ACT, never assume that two lines are parallel or perpendicular just because they look that way in a diagram. If the lines are parallel or perpendicular, the ACT will tell you so. (Perpendicular lines can be indicated by a little square located at the place of intersection, as in the diagram above.)

Equation of a Line

We've already shown you how to find the slope of a line using two points on the line. It is also possible to find the slope of a line using the equation of the line. In addition, the equation of a line can help you find the x- and y-intercepts of the line, which are the locations where the line intersects with the x- and y-axes. This equation for a line is called the slope-intercept form:

$$y = mx + b$$

where m is the slope of the line, and b is the y-intercept of the line.

Finding the Slope Using the Slope-Intercept Form

If you are given the equation of a line that matches the slope-intercept form, you immediately know that the slope is equal to the value of m. However, it is more likely that the ACT will give you an equation for a line that doesn't exactly match the slope-intercept form and ask you to calculate the slope. In this case, you will have to manipulate the given equation until it resembles the slope-intercept form. For example,

What is the slope of the line defined by the equation $5x + 3y = 6$?

To answer this question, isolate the y so that the equation fits the slope-intercept form.

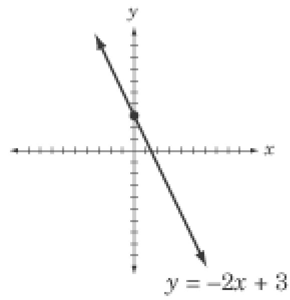
$$5x + 3y = 6$$
$$3y = -5x + 6$$
$$y = -\frac{5}{3}x + 2$$

The slope of the line is $-5/3$.

Finding the Intercepts Using the Slope-Intercept Form

The y-intercept of a line is the y-coordinate of the point at which the line intersects the y-axis. Likewise, the x-intercept of a line is the x-coordinate of the point at which the line intersects the x-

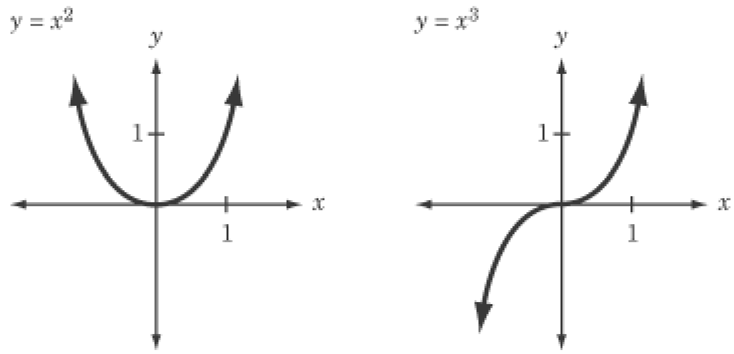
axis. In order to find the y-intercept, simply set $x = 0$ and solve for the value of y. To find the x-intercept, set $y = 0$ and solve for x.
To sketch a line given in slope-intercept form, first plot the y-intercept, and then use the slope of the line to plot another point. Connect the two points to form your line. In the figure below, the line $y = -2x + 3$ is graphed.



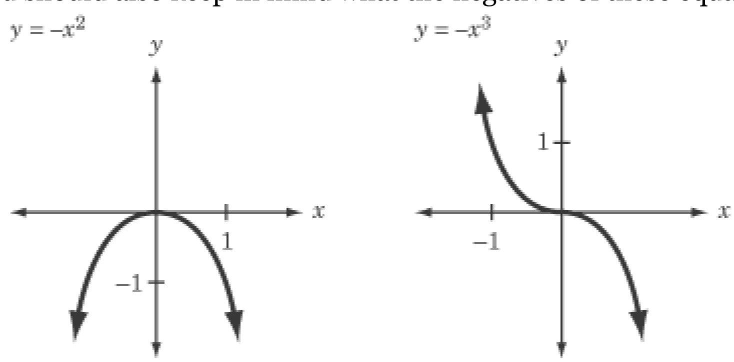
Since the slope is equal to −2, the line descends two units for every one unit it moves in the positive x direction. The y-intercept is at 3, so the line crosses the y-axis at (0,3).

Graphing Equations

For the ACT Math Test, you should know how the graphs of certain equations look. The two equations that are most important in terms of graphing are $y = x^2$ and $y = x^3$:



If you add lesser-degree terms to the equations, these graphs will shift around the origin but retain their basic shape. You should also keep in mind what the negatives of these equations look like:

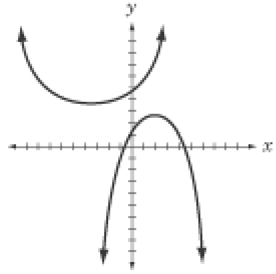


Conic Sections

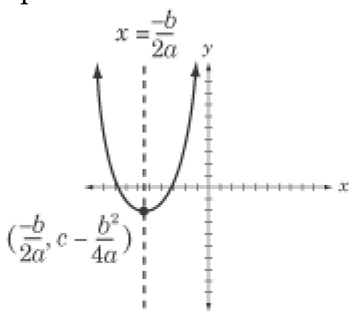
Occasionally, the ACT will test your knowledge of parabolas, circles, or ellipses. These topics do not regularly appear on the ACT, but it still pays to prepare: if these topics do appear, getting them right can separate you from the crowd.

Parabolas

A parabola is a "U"-shaped curve that can open either upward or downward.



A parabola is the graph of a quadratic function, which, you may recall, follows the form $ax^2 + bx + c$.
The equation of a parabola gives you quite a bit of information about the parabola.



1. The vertex of the parabola is $(-b/2a, c - b^2/4a)$.
2. The axis of symmetry of the parabola is the line $x = -b/2a$.
3. The parabola opens upward if $a > 0$, and downward if $a < 0$.
4. The y-intercept is the point $(0, c)$.

Circles

A circle is the collection of points equidistant from a given point, called the center of the circle. Circles are defined by the formula:

$$(x - h)^2 + (y - k)^2 = r^2$$

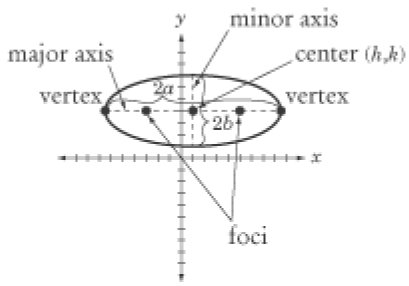
where (h,k) is the center of the circle, and r is the radius. Note that when the circle is centered at the origin, $h = k = 0$, so the equation simplifies to:

$$x^2 + y^2 = r^2$$

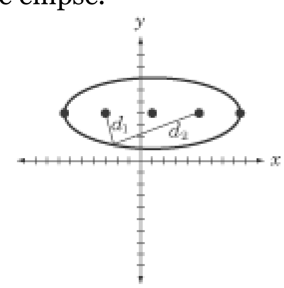
That's it. That's all you need to know about circles in coordinate geometry. Once you know and understand this equation, you should be able to sketch a circle in its proper place on the coordinate system if given its equation. You should also be able to figure out the equation of a circle given a picture of its graph with coordinates labeled.

Ellipses

An ellipse is a figure shaped like an oval. It looks like a circle somebody sat on, but it is actually a good deal more complicated than a circle, as you can see from all the jargon on the diagram below.



The two foci are crucial to the definition of an ellipse. The sum of the distances from the foci to any point on the ellipse is constant. To understand this visually, look at the figure below. The quantity $d_1 + d_2$ is constant for each point on the ellipse.



The line segment containing the foci of an ellipse with both endpoints on the ellipse is called the major axis. The endpoints of the major axis are called the vertices. The line segment perpendicularly bisecting the major axis with both endpoints on the ellipse is the minor axis. The point midway between the foci is the center of the ellipse. When you see an ellipse, you should be able to identify where each of these components would be.
The equation of an ellipse is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where *a*, *b*, *h*, and *k* are constants. With respect to this formula, remember that:

1. The center of the ellipse is (h,k).
2. The length of the horizontal axis is $2a$.
3. The length of the vertical axis is $2b$.
4. If $a > b$, the major axis is horizontal and the minor axis is vertical; if $b > a$, the major axis is vertical and the minor axis is horizontal.

http://www.sparknotes.com/testprep/books/act/chapter2.rhtml